An account of a Book. *Methodus Figurarum lineis rectis & curvis comprehensarum quadraturas determinandi Authore* J. Craige. 4to. Londini 1685..

THE great use of drawing the Tangents of Curve Lines, has made the most famous amongst the Modern Mathematicians endeavour to find out General Methods of finding the Tangents of Curve Lines, as may be seen from the several ways invented by *Des Cartes*, *Monsieur Fermat*, *Slusius*, Dr. *Barrow*, Dr. *Wallis*, *Tschurnehuys*, and *Leibnitius*; But as yet none has attempted to invert this problem generally, that is, having the Tangent to find the Curve Line whose tangent it is. Therefore the Author of this Treatise perceiving that the doing of this would give a General Method of determinating the Quadrature of any Curvilinear space, has laid down a rule for inverting *Slusius* his method mentioned in the *Philosophick Transactions Num.* 90. He has illustrated his Method of Quadratures by several Figures which have been already considered by Geometers. As for the Circle & Hyperbola, he asserts that their indefinite Quadratures are impossible, and therfore in these & such like cases, he expresses the Area by an infinite Series, which is easily done by his Method, except the Series consist of irrational termes, for in these he has recourse to *Leibnitius* his method of finding Tangents, where the Calculation will be more tedious. By his resolving the Area of the Hyperbola into an infinite series, he comes to the same expression with that of *N. Mercator*: And in measuring the Zone of a Circle, his expression falls in with that invented by Mr. *Isaac Newton*, as Mr. *David Gregory* relates in his Treatise. He has subjoyned a Method of measuring the Curve Superficies made by the rotation of any Curve upon its *Axis*, with a small Animadversion on the Method of *Quadratrices*, published in the *Acta Lipsiensia Eruditorum* of *October*, 168 .

Since the Publication of this Treatise, the Authour is pleased to make the following Addition.

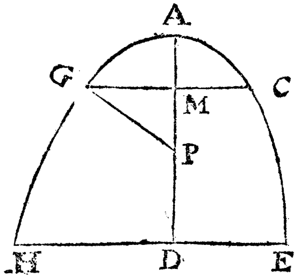
Additio ad Methodum Figurarum Quadraturas Determinandi. Autore *Johanne Craige*.

QUoniam omnium Figurarum Quadraturæ ex perfecta nostri primi problematis Solutione determinantur; propterea utile judicabam nonnulla addere, quæ solutionem meam non modo plenius illustrant sed omnino perficiunt: non adeo tamen sunt obscura, quin facile quisquam in istiusmodi rebus versatus, ex iis quæ jam exposui, omnia supplere possit. Problema sic se habet. Data expressione Analytica lineæ inter ordinatam & Curvæ perpendicularem designatæ, Invenire æquationem naturam illius Curvæ definientem. Hoc problema tres casus includit. 1. Cum expressio istius lineæ talis est, qualis a vulgaribus tangentium Methodis exhibetur. 2. Cum ad simpliciorem reducitur, facta divisione numeratoris & denominatoris per communem simplicem divisorem. 3. Cum expressio fit simplicissima, dividendo per divisorem compositum. Duos priores casus Regula, prout eam explicui, universaliter comprehendit; superest tantum ut ostendam quo pacto tertium pariter casum comprehendat.

Postquam expressio data per y *multiplicatur, apponantur omnes termini qui sub maximo continentur (Terminorum magnitudinem e dimensionibus quantitatis* y *mensurans) & connectantur signo affirmativo vel negativo, prout libuerit; adæquentur omnes illi termini (prius in coefficientes incognitas multiplicati) Quadrato quantitatis per* x *designatæ: eritq; inde resultans æquatio quæsita, vel quæsitam includet; & determinationes coefficientium terminos æquationem constituentes a reliquis distinguent.*

Sit

Sit in apposito schemate abscissa AM$=y$, *ordinata* MC$=z$, *& Curvæ* ACE *proprietas* $z^2 = \frac{a^2y^2+y^4}{p^2}$, *& invenienda sit quadratura Areæ a lineis rectis & illa Curva comprehensæ. Quærenda est alia Curva* AGH, *in qua* PM$=$ $\sqrt{\frac{a^2y^2+y^4}{p^2}} = z$; *ubi* PG *Curvæ quæsitæ perpendicularem, &* MG$=x$ *illius ordinatam denotat. Cumq; hæc expressio lineæ* PM *in* y *multiplicata contineat sextam quantitatis* y *dimensionem, ideo appono omnes terminos sub illa sexta dimensione contentos, unde resultans æquatio est.*



$$\frac{na^6+ma^5y+la^4y^2+ha^3y^3+ka^2y^4+gay^5+fy^6}{p^2} = x^4.$$

Ex hac æquatione invenio valorem Lineæ PM. *quem comparo cum valore dato, unde*

$$PM = \frac{ma^5+2la^4y+3ha^3y^2+4ka^2y^3+5gay^4+6fy^5}{4p\sqrt{na^6+ma^5y+la^4y^2+ha^3y^3+ka^2y^4+gay^5+fy^6}} = \sqrt{\frac{a^2y^2+y^4}{pp}}.$$

auferantur fractiones & Signa radicalia, & determinentur coefficientes n, m, l &c: (*ut in* prob: 2 *tractatus nostri*) *rejectis iis quarum determinationes absurdum involvunt: & cæteræ, in quibus nihil tale contingit, æquationem constituent. Sic in exemplo proposito, erit* $f = \frac{4}{9}$, $k = \frac{12}{9}$, $l = \frac{12}{9}$, $n = \frac{4}{9}$. *sed dum* g *determino, invenio* $240g = 144g$, *quod absurdum involvit; & sic pro* h *comparatio erit* $16h = 16h$ *unde nullus illius valor, & pro* m *erit* $4m = 4m$, *quod itidem est absurdum: quapropter termini a quantitatibus* g, h, m *affecti ad æquationem non pertinebunt; unde reliqui a literis* n, l, k, f *affecti æquationem naturam Curvæ definientem constituent. sc.* $\frac{4a^6+12a^4y^2+12a^2y^4+4y^6}{9p^2} = x^4$,

$=x^4$, *adeoq;* AMC$=\sqrt{\frac{a^6+3a^4y^2+3a^2y^4+y^6}{6p^2}}=\frac{x^2}{2}$. *Exem.* 2

Sit Curvæ Lineæ A C E *talis proprietas* $z^2=\frac{qy^2+y^3}{q}$ *& inveniendà sit Quadratura spatii* A M C. *Quærenda est Curva* AGH *in qua sit* P M$=\sqrt{\frac{qy^2+y^3}{p}}=$Z *& quoniam hic valor in* y *multiplicatus continet quintum quantitatis* y *dimensionem, apponantur omnes termini sub illa quinta dimensione, & æquentur quadrato quantitatis per* x *designatæ; unde æquatio resultans est.*

$\frac{nq^5+mq^4y+lq^3y^2+kq^2y^3+hqy^4+fy^5}{p}=x^4$. *atq; sola coefficientis* (m) *determinatio absurdum involvet, eruntq; reliquæ,* $n=\frac{64}{225}$, $l=-\frac{16}{15}$, $k=-\frac{16}{45}$, $h=\frac{16}{15}$, $f=\frac{16}{25}$, *unde æquatio curvam quæsitam definiens est.*

$\frac{64q^5}{225p}-\frac{16q^3y^2}{15p}-\frac{16q^2y^3}{45p}+\frac{16qy^4}{15p}+\frac{16y^5}{25p}=x^4$ *adeoq;*

AMC$=\sqrt{\frac{16q^5}{225p}-\frac{4q^3y^2}{15p}-\frac{4q^2y^3}{45p}+\frac{4qy^4}{15p}+\frac{4y^5}{25p}}=\frac{xx}{2}$

Exem. 3. *Inuenienda sit Quadratura spatii* AMC, *definita natura Curvæ* ACE *hac Æquatione* $z^2=\frac{a^3}{4y+4a}$. *Quæratur alia Curva* AGH, *in qua* PM$=\sqrt{\frac{a^3}{4y+4a}}=$z. *Ex præmissis constat Æquationem primam fore* $\frac{na^3y^2+ma^4y+16a^5}{4a+4y}=4x$ *& determinationes Coefficientium* $n=16$, $m=32$, $l=16$. *Quibus substitutis, erit æquatio* $\frac{16a^3y^2+32a^4y+16a^5}{4y+4a}=x^4$ $=4a^3y+4a^4$: *adeoq;* AMC $=\sqrt{a^3y+a^4}=\frac{1}{2}x^2$.

Notatu dignissimum est, has tres (sicut infinitas alias) Quadraturas abscissæ AM *(seu* y *) non convenire. Quoniam in istiusmodi*

modi Figuris, simplicissima Areæ expressio huic portioni non respondet: attamen Quadratura abscissæ conveniens exinde parvo labore deducitur. Ut in Exem: 3. *ubi Area est* $\sqrt{a3y+a4}$; *fiat* $y=0$, *&* *erit Area* $\sqrt{a4}=a2$, *& subducatur hæc ex generali, proveniet Quadratura portionis abscissæ respondentis, sc.* $\sqrt{a3y+a4}-a2$. *Quam observatiunculam mihi primus significavit Vir celeberrimus D. Isaacus Newton.*

Tentetur jam idem processus in Circulo ACE, *cujus diameter sit* r, *ac proinde* $Z=\sqrt{ry-y2}$, *Quærenda est Curva* AGH *in qua* $PM=\sqrt{ry-y2}=z$, *sed ex dictis constat æquationem primam fore* $nr4+mr3y+tr2y2+hry3-ky4=x4$: *& singulæ coefficientium determinationes erunt impossibiles; adeoq; nulla datur Curva* AGH *in qua* $PM=\sqrt{ry-y2}$, *ac proinde Circuli Quadratura indefinita est impossibilis. Fieri tamen potest ut sit aliqua hujusmodi Curva* AGH, *sed ex earum numero, quas post* Cartesium *Mechanicas Geometræ communiter appellant: sed quia harum usus non libenter admittunt Mathematici, præstat hujusmodi Quadraturas per series infinitas exhibere.*

Benevole Lector

Ob inopiam Typorum Numeralium minusculorum, qui ad designandas quantitatum potestates supra Symbola dextrorsum apponi solent, festinante prælo, Typographus paulo majoribus usus est in eadem linea immediate sequentibus; ubicunq; itaq; offenderis a3, *vel* x2, *&c. cubum vel quadratum, &c.* e *quantitate, cui suffigitur numerus, intelligas.*

LONDON,

Printed by *Joſeph Streater*, and are to be ſold by *Samuel Smith*, at the *Princes Arms* in St. *Pauls* Church Yard.